Probability is the measure of the likelihood that an event will occur. It is a number between zero and one.

Outcomes and evends

An outcome is a single possible result of a random experiment (also called a procedure).

An event is any collection of outcomes.

A simple event is an event consisting of just one outcome.

Ex) Consider the random experiment of going to a baseball game and recording the score between two teams as an outcome.

possible outcome: 3 to 7
another outcome: 4 to 5
An event: a tie score (stands for a collection of possible outcomes e.g. 0 to 0, 1 to 1, 2 to 2, etc)

Note that "Was it a tie or not?" question can be answered with a yes or no based on the outcome of a random experiment. & how you judge if something is an event.

Another event: a shutout (a collection of scores for which one team scored zero points)
ex) 1:0, 13:0, 0:5

A simple event: The score was 5 to 8.

This simple event consists of only one outcome.

Ex) income levels for a family

An event: income is between \$30,000 and \$40,000.

A simple event: income was exactly \$34,950.32.

There is little that can be said about what will happen on a single run of a random experiment. However, when the experiment is repeated many times, some patterns may emerge involving the proportion of times an event happens.

Ex) Flip coins. five flips: possible outcome HHHHT

1000 flips: roughly half heads, half tails __relative frequency__ of an event is equal to the number of times the event happened divided by the number of runs of the random experiment.

$$\text{rel freq}(A) = \frac{\#(A)}{n}$$

Ex) 20 coin flips: 11 H, 9 T | A: get a head
B: get a tail

$\text{rel freq}(A) = \frac{11}{20}$; $\text{rel freq}(B) = \frac{9}{20}$

Law of large numbers

Every event has a special number (called its probability) such that if the random experiment is repeated a __large number of times__, then the relative frequency of the event will be close to this number.

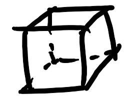
Ex) coin tossing: $P(A) = P(B) = 0.5$) this is __not__ exactly true for monetary coins.

__Probability__
- always between 0 and 1
- near 1 indicates a nearly certain event
- near 0.5 indicates an event is as likely to happen as it is not

1 — certain
likely
0.5 — 50-50 chance
unlikely
0 — impossible

For the case of equally likely outcomes:

$$P(\text{event}) = \frac{\#\text{ of outcomes in the event}}{\text{total }\#\text{ of outcomes}}$$

Ex) consider a six sided die  faces labeled 1, 2, .., 6

$P(\text{even number}) = \frac{3}{6} = 0.5$ [{2, 4, 6} out of {1, 2, 3, 4, 5, 6}]

$P(\text{draw 5 or 6}) = \frac{2}{6} = \frac{1}{3}$ | $P(\text{draw }7) = 0$
$P(\text{drawing 1-6}) = 1$

Ex) raffle amongst 653 people < 314 women, 339 men

$P(\text{woman wins raffle}) = \frac{314}{653} \approx 0.48$

Ex) Deck of 52 playing cards

$P(\text{being dealt 8 of clubs}) = \frac{1}{52}$

$P(\text{being dealt a heart}) = \frac{13}{52} = \frac{1}{4}$ (13 cards of 4 suits: hearts, spades, clubs diamonds).

$P(\text{being dealt an ace}) = \frac{4}{52} = \frac{1}{13}$

$P(\text{being dealt a five}) = \frac{4}{52} = \frac{1}{13}$

or A^c

<u>Complement</u> of event A: denoted by $\overline{A}$, consists of all outcomes in which the event A does not occur.

Ex) 1010 adults surveyd — 202 smokers
— rest non-smokers

$P(\text{smoker}) = \frac{202}{1010} = 0.2$

$P(\text{non-smoker}) = 1 - \frac{202}{1010} = 0.8$

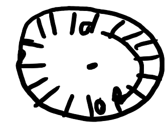
$$\boxed{P(\text{not } A) = P(\bar{A}) = 1 - P(A)}$$

odds against or in favor of

actual odds against event A occuring are the ratio $\frac{P(\bar{A})}{P(A)}$.

actual odds in favor of event A occuring are the ratio $\frac{P(A)}{P(\bar{A})}$.

Ex) play roulette (#s 1 to 38)



If you bet \$5 on the number 13 in roulette your probability of winning is $\frac{1}{38}$.

odds against the outcome of 13:

$P(13) = \frac{1}{38}$

$P(\text{not } 13) = 1 - \frac{1}{38} = \frac{37}{38}$

odds against 13: $\frac{P(\text{not } 13)}{P(13)} = \frac{37/38}{1/38} = \frac{37}{1}$

reported as "37:1."

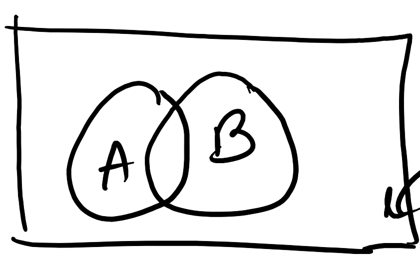
compound event : any event combining two or more simple events.

P(A or B) = P(A) + P(B) − P(A and B)

additive rule (useful for evaluating more complicated probabilities)

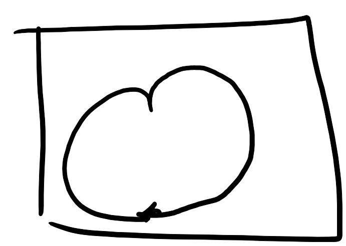
The probability of the union of events A and B is the sum of the probability of events A and B minus the probability of intersection of events A and B.

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$



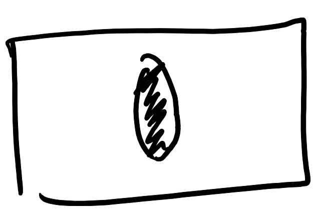
sample space

$A \cup B$



union of A and B

intersection of A and B ($A \cap B$)



Events A and B are mutually exclusive if $A \cap B$ contains no simple events.

Ex) consider tossing two fair coins. Find the probability of observing at least one head.

A: { observe at least one head } ← it may not be easy to get p(A). (in this case it is).

notice p(A) is what you want!

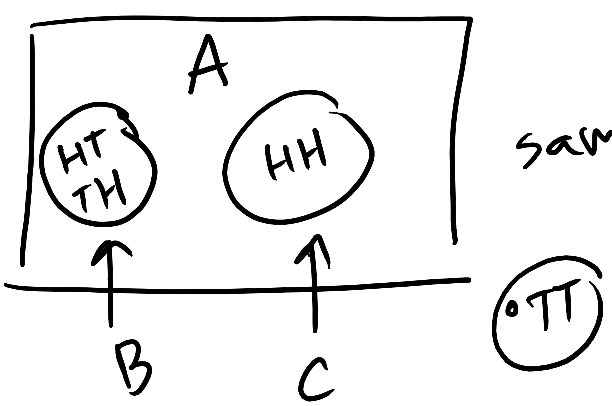
want to express A as a union of two events.

B: { observe exactly one head } $p(B) = \frac{2}{4} = \frac{1}{2}$

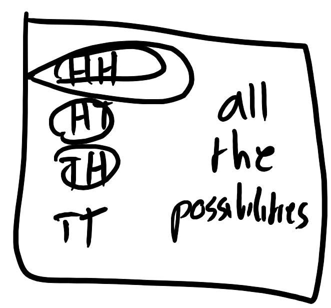
C: { observe exactly two heads } $p(C) = \frac{1}{4}$

$A = \underline{B \cup C}$ (event A is the union of B and C)

$B \cap C$ (intersection) $= \phi$



sample space :



$$P(A) = P(B \cup C) = P(B) + P(C) - P(B \cap C)$$

$$= P(B) + P(C) - 0 = \frac{1}{2} + \frac{1}{4} - 0 = \frac{3}{4}$$

M: exactly one head: HT
TH

HT : event $\hat{A}$

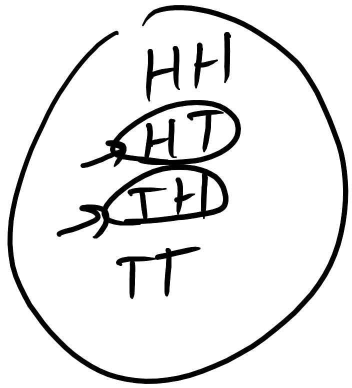
TH : event $\hat{B}$

$$P(M) = P(\hat{A} \cup \hat{B}) = P(\hat{A}) + P(\hat{B}) - P(\hat{A} \cap \hat{B}).$$

$$\boxed{P(\hat{A} \cap \hat{B}) = 0}$$

$$= \frac{1}{4} + \frac{1}{4} - 0 = \frac{1}{2}$$

entire sample space



directly:

$$P(M) = \frac{2}{4} = \frac{1}{2}$$