#1

5 cards drawn from 52 card deck

$$\# \text{ of combinations} = \binom{52}{5} = {}_{52}C_5$$

(a) $$P(4 \text{ aces}) = \frac{({}_4C_4)({}_{48}C_1)}{{}_{52}C_5} = \frac{1}{54{,}125}$$

picking 4 out of 4 aces (all aces) and 1 card out of remaining $52-4=48$ cards.

(b) $$P(4 \text{ aces and } 1 \text{ king}) = \frac{({}_4C_4)({}_4C_1)}{{}_{52}C_5}$$

$$= \frac{1}{649{,}740}$$

Pick all available aces and 1 king out of 4

(c) P(3 tens and 2 jacks)

$$= \frac{({}_4C_3)({}_4C_2)}{{}_{52}C_5} = \frac{1}{108,290}$$

picking 3 of 4 tens and 2 of 4 jacks in the deck.

(d) P(nine, ten, jack, queen, king) =

$$= \frac{({}_4C_1)({}_4C_1)({}_4C_1)({}_4C_1)({}_4C_1)}{{}_{52}C_5} = \frac{64}{162,435}$$

for each card pick 1 out of 4 available; order doesn't matter.

(e) P(at least one ace)

$$= 1 - P(\text{no aces}) = 1 - \frac{{}_{48}C_5}{{}_{52}C_5} = \frac{18,472}{54,145}$$

complement event easier to evaluate

There are $52-4=48$ cards which are not aces.

Section 4-5

22. a. $\frac{860}{950}$ or 0.905

    b. $\frac{860}{866}$ or 0.993

    c. The results are different

33. a. $\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \ldots \cdot \frac{341}{365} = 0.431$

    b. $1 - 0.431 = 0.569$

Section 4-6

3. Because repetition is allowed, numbers are selected with replacement, so neither of the two permutation rules applies. The fundamental counting rule can be used to show that the number of possible outcomes is $10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$, so the probability of winning is $\frac{1}{10{,}000}$.

9. The number of combinations is $\frac{27!}{(27-12)!12!} = 17{,}383{,}860$. Because that number is so large, it is not practical to make a different CD for each possible combination.

10. $\frac{1}{52} \cdot \frac{1}{51} + \frac{1}{52} \cdot \frac{1}{51} = \frac{1}{1326}$

12. $\frac{8!}{(8-3)!} = 336$

14. $\frac{10!}{3!3!2!} = 50{,}400$

16. $\frac{53!}{(53-6)!6!} = 22{,}957{,}480$. The probability is $\frac{1}{22{,}957{,}480}$