HW #12

(1) see posted notes on hypothesis tests for the solution.

(2) $\sigma^2 = 0.25$ normal population

$n = 9$ (sample size)

$H_0: \mu = 1$ vs $H_1: \mu = 2$

Decision rule: reject H_0 when $\bar{X} > 1.38$.

Notice that this decision rule is reasonable. Thus, we expect the probabilities of type I and type II errors to be small.

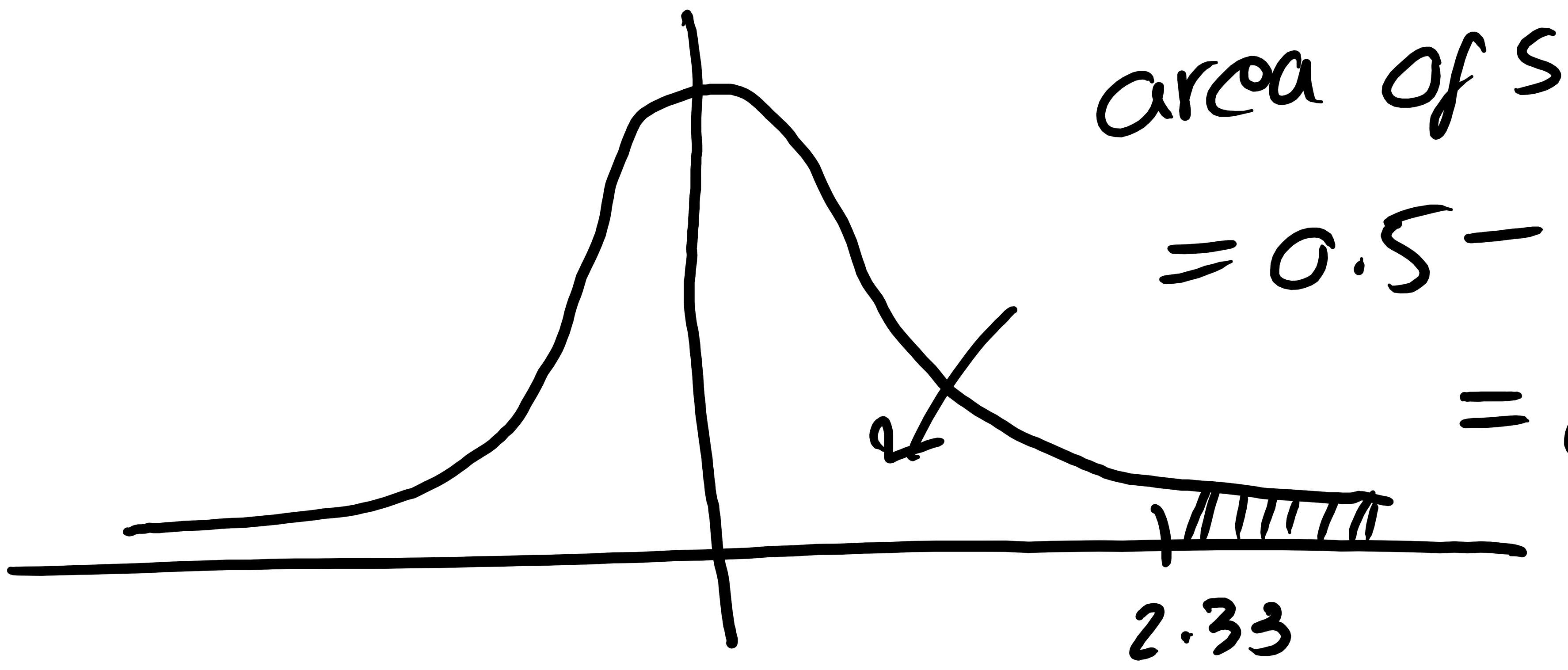
$\alpha = P(\text{type I error}) = P(\text{reject } H_0 \text{ when } H_0 \text{ true})$

$= P(\bar{X} > 1.38 \text{ when } \mu = 1)$

$$= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \frac{1.38 - 1}{.5/\sqrt{9}} \right)$$

$$= P(z > 2.33) = .01 \text{ (from table)}$$

since



area of shaded region
$= 0.5 - P(0 < z < 2.33)$
$= 0.5 - 0.49 = 0.01$

Notice that $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$ but not by the CLT since $n < 30$. It is because we know that the population is normal so any samples drawn from it would be normally distributed and so $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ is std normal (mean zero and std deviation 1).

$\beta = P(\text{type II error}) = P(\text{accept } H_0 \text{ when } H_0 \text{ false})$

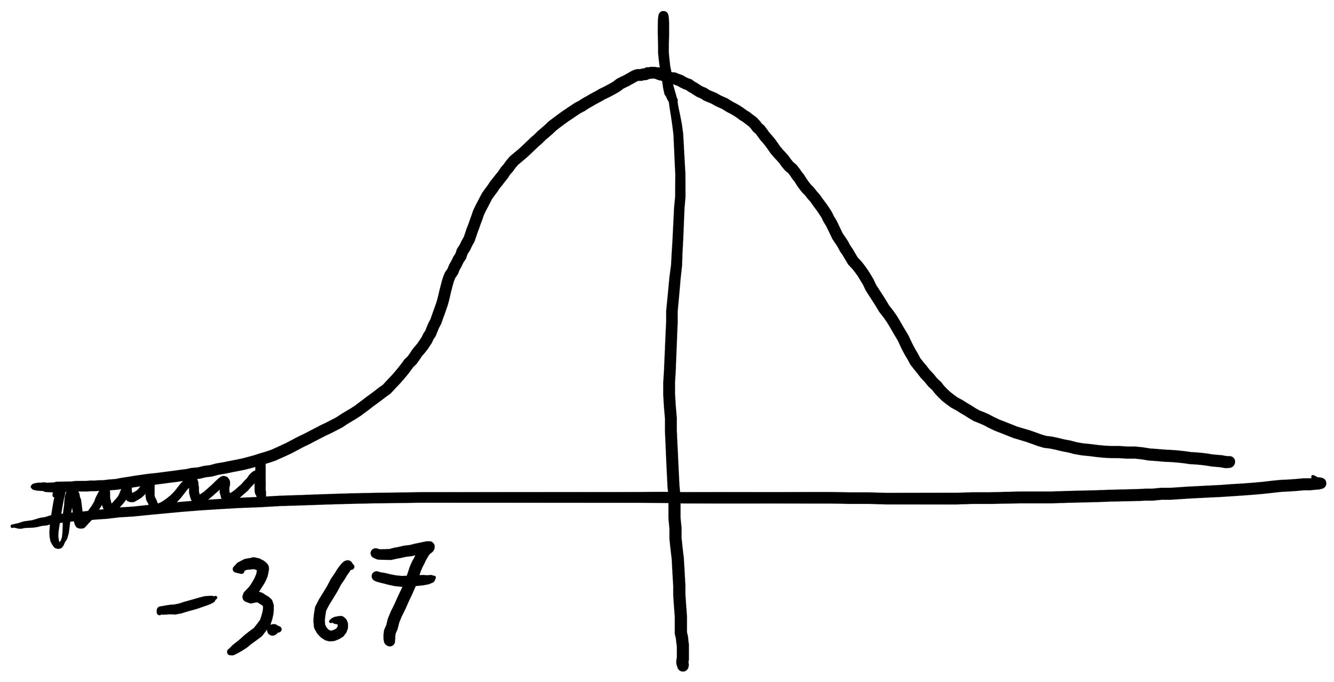
$= P(\text{accept } H_0 \text{ when } H_1 \text{ true})$

$= P(\bar{X} \leq 1.38 \text{ when } \mu = 2)$

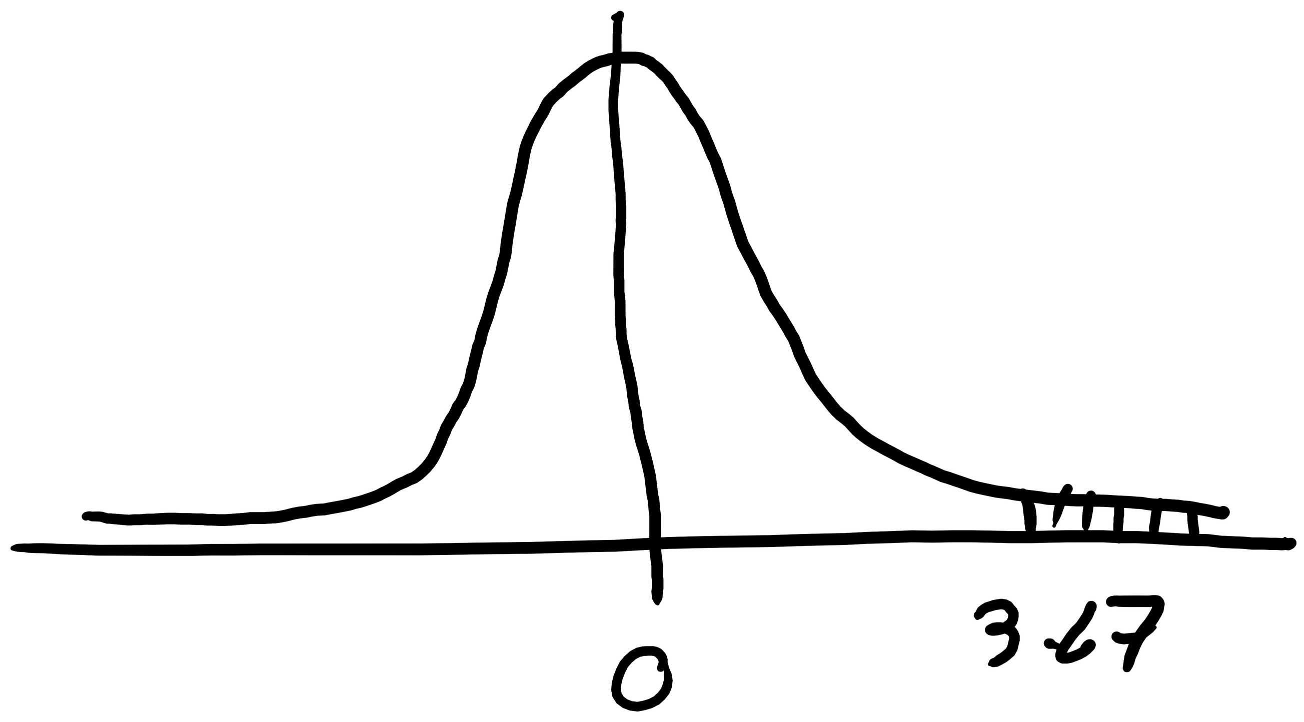
$$= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{1.38 - 2}{.5/3} \right)$$

$$= P(Z \leq -3.67)$$

≈ 0.001



This area is the same as this one on the right



$$P(Z \leq -3.67) = P(Z \geq 3.67) = 0.5 - P(0 < Z < 3.67)$$
$$= 0.5 - 0.499 = 0.001$$

As expected, both type I and type II errors are small.