

**MATH 21-01 (Introductory Statistics), HW 12 (100 points). Due: 12/06/2016 in class.**

**Not from textbook (30 pts)**

1. Suppose a coin collector has a rare coin with probability of head flip equal to  $\frac{3}{4}$  instead of  $\frac{1}{2}$ . Upon misplacing the coin, he searches for look alike coins and tries to determine if one of them happens to be the rare coin he wants to find. Let  $p$  denote the probability of heads for a coin he tests. For a given coin, the collector sets up the following hypothesis test:

$$H_0 : p = \frac{3}{4} \quad ; \quad H_1 : p = \frac{1}{2}$$

The null hypothesis  $H_0$ , is that the coin is the rare coin, while the alternate hypothesis  $H_1$ , is that the coin is the regular one.

The coin the collector wants to test is flipped 100 times. Suppose the following decision rule is used to judge if or not to accept or reject the null hypothesis.  $H_0$  is rejected if 100 flips yield fewer than 65 heads (and the conclusion is that the coin is the regular one).  $H_0$  is accepted if 100 coin flips yield at least 65 heads. Answer the following questions for this hypothesis test, utilizing the normal approximation to the Binomial distribution:

- (A) Find the probability of a type I error. That is, find  $P(\text{test will reject } H_0 \text{ when } H_0 \text{ is true})$ .
- (B) Find the probability of a type II error. That is, find  $P(\text{test will accept } H_0 \text{ when } H_0 \text{ is false})$ .
- (C) If the test is repeated for a large number of coins, approximately how accurate would the test be?

2. Let a population be normal with known variance  $\sigma^2 = 0.25$  and unknown mean  $\mu$ . Suppose a hypothesis test is set up for the population mean  $\mu$  where:

$$H_0 : \mu = 1 \quad \text{versus} \quad H_1 : \mu = 2$$

Let  $\bar{x}$  denote the mean of a random sample of 9 values from the population. Suppose the following decision rule is used: *Reject  $H_0$  when  $\bar{x} > 1.38$* . Find the probabilities of type I and type II errors, defined as in problem 1, for this test.

**From textbook (70 pts)**

Section 7.4: 7, 14

Section 8.2: 3, 5, 6, 7, 13, 14, 15, 16, 25, 26, 27

Section 8.3: 10, 12