

MATH 21-01 (Introductory Statistics, Voronin, S.), Final Exam (150 points).

You may use a calculator and one page of notes (both sides). Clearly state your answer to each question. Please show your work for all problems, don't just write a numerical answer. Sufficient written reasoning and calculation work is required to obtain full credit. Partial credit will be awarded where possible. Start: 12:00 PM. End: 2:00 PM. December 19th, 2016.

Problem I, 15 pts

In a certain city, the probabilities are 0.86, 0.35, and 0.29 that a given home is outfitted with security system I, II, and with both systems, respectively.

- (a, 5 pts) Find the probability that a family home owns either system I, II, or both security systems.

Next, suppose for certain events A, B, C the probabilities are $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{7}$.

- (b, 10 pts) Calculate the probability of the event $A' \cap B'$.

Problem II, 15 pts

A small cell phone charger manufacturer suspects that the proportion of its defective units has increased from .01. To test the claim, an employee randomly selects 80 chargers and found 2 to be defective.

- (a, 7 pts) Conduct the test of hypothesis to test the claim at the $\alpha = 0.05$ level of significance. What is your decision rule? What is your conclusion?

Let a population be normal with known variance $\sigma^2 = .25$ and unknown mean μ . Suppose a hypothesis test is set up for the population mean μ where $H_0 : \mu = 1$ versus $H_1 : \mu = 2$. Nine values from the population are sampled. Suppose the following decision rule is used: Reject H_0 when $\bar{x} > 1.38$.

- (b, 8 pts) What do Type I and Type II errors for this test represent? Find the probability of Type I and Type II error.

Problem III, 15 pts

Suppose two standard dice are rolled (each face has a number from 1 to 6) and the numbers shown on top for each dice are summed together.

- (a, 2 pts) What is the probability of getting a sum of 7 on the two rolls?

Suppose that X is a random variable counting the number of times a sum of 7 is attained in rolling two dice three times (three trials, two dice rolled at each trial).

- (b, 7 pts) Which probability distribution does X follow? Find the probability of getting at least one sum of 7 in three trials.

Next, suppose that the probability that an individual suffers a bad reaction from a vaccine is 0.005. The vaccine is given to 1000 people.

- (c, 6 pts) Find the probability that more than 2 people will suffer ill effects of the vaccine, using the Poisson distribution as an approximation. You may leave your answer in terms of exponentials.

Problem IV, 15 pts

Suppose the same spinach company we looked at previously also produces fresh kale in bags and reports the following probability distribution of dead bug remains found per bag:

$$P(X = 0) = 0.65, \quad P(X = 1) = 0.18, \quad P(X = 2) = .10, \quad P(X = 3) = .07$$

where X is a random variable recording the number of dead bug remains in a bag.

- (a, 15 pts) Using the CLT for the sample mean, find the probability of finding more than 50 dead bug remains in 50 bags of the kale.

Problem V, 15 pts

An ATV manufacturer wants to determine the mean horsepower of a motor it has developed, operating at different elevations from 0 to 2 km altitude above sea level. The manufacturer conducts 10 tests in different locations. It obtains 65 as the mean horsepower with standard deviation of 8 for the sample.

- (a, 15 pts) Construct 90% and 95% confidence intervals for the true mean horsepower produced by the motor.

Problem VI, 15 pts

A company seeks to determine the mean operating time of its drone battery. It has conducted 50 test flights and found a mean operating time of 25 minutes before the battery dies. Suppose the standard deviation of the operating times was $s = 5$ and we use s as an estimate of the true population standard deviation σ (that is, assume σ is known).

- (a, 15 pts) Estimate the probability that when the sample mean \bar{x} of the flight times is used as an estimator of the population mean μ , the absolute estimation error will be less than 3 minutes.

Problem VII, 15 pts

Suppose the same company above does another 12 flight tests in arctic conditions. They obtain the following set of operating time measurements in minutes:

$$[18, 22, 25, 8, 19, 32, 20, 18, 17, 21, 22, 18]$$

- (a, 10 pts) Find the five number summary of this data set and the IQR. Are there any outliers based on the IQR characterization?
- (b, 5 pts) Draw a modified boxplot for the data.

Problem VIII, 15 pts

Suppose a fair coin is tossed 500 times. Let X count the number of heads in the 500 tosses.

- (a, 5 pts) Which distribution does X follow? What is the mean and variance of X ?
- (b, 15 pts) Using a normal distribution approximation, find the probability that the number of heads will not differ from 250 by more than 12.

Problem IX, 15 pts

Suppose that two brands A and B of filtered milk are compared. The manufacturers claim that the content of fat in both milks per glass is the same. 50 glasses of brand A are sampled and the fat content is found to be 10 grams with a std deviation of 1 g. Then, 60 glasses of brand B are sampled and the fat content is found to be 12 g with a std deviation of 1.5 g. Suppose we use the sample std deviations as estimators of the population std deviations (that is assume σ_A, σ_B are known).

- (a, 15 pts) Set up and conduct a hypothesis test for testing the hypothesis that the fat content of brand B exceeds that of brand A at the $\alpha = 0.01$ confidence level. What is your conclusion? What is the probability of a Type I error?

Problem X, 15 pts

Suppose 5 cards are drawn from a standard 52 card deck (without replacement). Below, you may leave your answer in terms of combinations or permutations.

- (a, 5 pts) Find the probability that there are 3 aces amongst the 5 cards.
- (b, 5 pts) Find the probability of getting 3 kings and 2 queens.

For two events A and S (someone being attacked and your friend being sighted by that black bear on campus from exam 1, respectively), let $P(S) = 0.1$, $P(A) = 0.2$, and $P(S \cap A) = 0.05$.

- (c, 5 pts) Calculate $P(A|S)$. Are A and S independent?