Solutions for exam #2

(I) (a) Binomial, discrete

(b) $P(X>3) = 1 - P(X \le 3)$

$= \left[1 - \left[P(X=0) + P(X=1) + P(X=2) + P(X=3)\right]\right]$

For each prob we use Poisson approx.

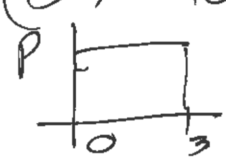
$\lambda = np = 2000(.001) = 2$

$$P(X_{poisson}=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$P(X_{binom}>3) \approx \left[1 - \sum_{k=0}^{3} \frac{\lambda^k e^{-\lambda}}{k!}\right] \quad \text{(approx via poisson)}$$

(c) Continuous (uniform distribution)

(d) 40 days $= \frac{40}{30} = \frac{4}{3}$ months



$3p = 1 \Rightarrow p = \frac{1}{3}$ $\quad P(X_{unif} \le 40) = \frac{4}{3} \cdot \frac{1}{3} = \boxed{\frac{4}{9}}$

same prob of failure between 0 and 3 months

II) (a)

$$P(X=k)=\binom{10}{k}\left(\frac{1}{2}\right)^{k}\left(\frac{1}{2}\right)^{7-k}$$ [redacted]

$$P(3 \le X_{binom} \le 6)=\sum_{k=3}^{6}\binom{10}{k}\left(\frac{1}{2}\right)^{k}\left(\frac{1}{2}\right)^{7-k}$$

$\simeq 0.7734$ [redacted] [redacted]

(b) now we approximate the probability with the Normal distribution:

$$P(3 \le X_{binom} \le 6) \le P(2.5 \le X_{normal} \le 6.5)$$

$$=P\left(\frac{2.5-np}{\sqrt{npq}} \le Z_{normal\ std} \le \frac{6.5-np}{\sqrt{npq}}\right)$$ [redacted]

$$E[X_{binom}]=np=10\left(\frac{1}{2}\right)=5$$ [redacted]

$$\sigma_{X_{binom}}=\sqrt{npq}=\sqrt{10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} \doteq 1.58$$

$$P(3 \le X_{binom} \le 6) \simeq P\left(\frac{2.5-5}{1.58} \le Z \le \frac{6.5-5}{1.58}\right)$$

$$=P(-1.58 \le Z \le 0.95) = P(-1.58 \le Z \le 0) + P(0 \le Z \le 0.95)$$ [redacted]

$= 0.4429 + 0.3289 \approx \boxed{0.7718}$

values are almost the same.

(III) (a) $\mu = 72, \sigma = 24$

(notice that these are population values for your driving)

$$P(40 \le X \le 60) = P\left(\frac{40-\mu}{\sigma} \le Z \le \frac{60-\mu}{\sigma}\right)$$

$$= P\left(\frac{40-72}{24} \le Z \le \frac{60-72}{24}\right)$$

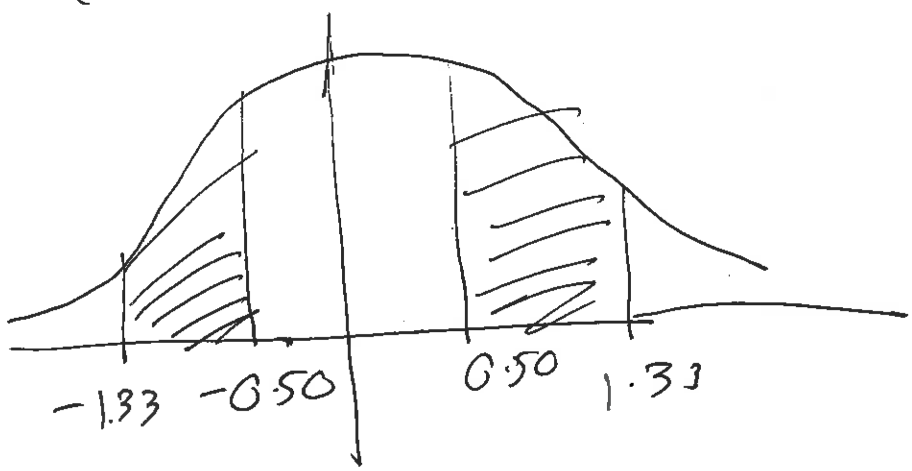
3 (a) $\mu = 72$, $\sigma = 24$

population values for your driving

$P(40 < x < 60) =$

$= P\left(\frac{40-72}{24} < z < \frac{60-72}{24}\right)$

$= P(-1.33 < z < -0.50)$



$= P(0.50 < z < 1.33)$

$= P(0 < z < 1.33) - P(0 < z < 0.50)$

$= .4082 - .1915 = \boxed{0.21670}$

(b) $\bar{X}=65$, $\sigma=20$ (take $\sigma=5$)

$n=100$

$$P(|\bar{x}-\mu|<7)=?$$

$$z=\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}\sim N(0,1) \quad ; \quad P(|z|<z_{\alpha/2})=1-\alpha$$

$$P\left(\frac{|\bar{x}-\mu|}{\sigma/\sqrt{n}}<z_{\alpha/2}\right)=1-\alpha$$

$$\Rightarrow P\left(|\bar{x}-\mu|<z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)=1-\alpha$$

$$\Rightarrow z_{\alpha/2}\frac{\sigma}{\sqrt{n}}=7 \Rightarrow z_{\alpha/2}\frac{20}{\sqrt{100}}=z_{\alpha/2}\cdot 2=7$$

$$\Rightarrow z_{\alpha/2}=\frac{7}{2}$$

$$P(|z|<z_{\alpha/2})=2P(0<z<z_{\alpha/2})=2P\left(0<z<\frac{7}{2}\right)$$

$$=1-\alpha=2(.4998)=\boxed{0.9996}$$

$$P(0<z<3.5)=A(3.5)=.4998 \Rightarrow \cancel{2x}\ 2\times.4998=$$

(a) 4

$P(X=0)=0.70$

$P(X=1)=0.20$

$P(X=2)=0.07$

$P(X=3)=0.03$

probability distribution since $\sum_{k=0}^{3} P(X=k)=1$

$E[X]=\sum x\,p(x)$

$=0(0.70)+1(0.20)+2(0.07)+3(0.03)$

$=0.43=m$

$\sigma^2=\sum (x-m)^2 p(x)$

$=(0-0.43)^2(0.70)+(1-0.43)^2(0.20)$

$+(2-0.43)^2(0.07)+(3-0.43)^2(0.03)$

$=\underbrace{0.565}_{=\sigma^2} \Rightarrow \boxed{\sigma \approx 0.752}$

(c) P(more than 50 scratches)

$= P(\bar{X} > 1)$ since there are $n=50$ pots

$$= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \frac{1-\mu}{\sigma/\sqrt{n}} \right) = P\left(Z > \frac{1-\mu}{\sigma/\sqrt{n}} \right)$$

$$= P\left(Z > \frac{1 - .43}{.752/\sqrt{50}} \right) = P\left(Z > \frac{.57}{0.106} \right)$$

$= P(Z > 5.377) \sim 0$ very small probability!

(5) $\{1.5, 5.5, 3.6, 3.5, -0.5\}$

(a)

$$\bar{x} = \frac{\sum x}{n} = 2.72$$

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$S^2 = \frac{1}{4} \left[(1.5-2.72)^2 + (5.5-2.72)^2 + (3.6-2.72)^2 + (3.5-2.72)^2 + (-0.5-2.72)^2 \right]$$

$= 5.24 \Rightarrow \boxed{S = 2.29}$

(b) For confidence interval, must use the t-distribution since n is small and pop. std dev. is not known.

$$\bar{x} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

$\alpha = 0.10 \Rightarrow \frac{\alpha}{2} = 0.05 \qquad n = 5 \Rightarrow n-1 = 4$

$t_{\alpha/2, n-1} = t_{0.05, 4} = 2.132$

$$\Rightarrow 2.72 - 2.132 \frac{2.29}{\sqrt{5}} < \mu < 2.72 + 2.132 \frac{2.29}{\sqrt{5}}$$

$\boxed{0.54 < \mu < 4.90}$ 90% CI for population mean