

MATH 21-01 (Introductory Statistics, Voronin, S.), Early Exam #2 (100 points).

You may use a calculator and one page of notes (both sides). Clearly state your answer to each question. Please show your work for all problems, don't just write a numerical answer. Sufficient written reasoning and calculation work is required to obtain full credit. Partial credit will be awarded where possible. Start: 9:30 AM. End: 10:20 PM. November 22nd, 2016.

Problem I, 20 pts

Suppose the probability that an individual will suffer a bad reaction from a vaccine is 0.001. The vaccine is administered to $n = 2000$ people.

- (a, 2 pts) Which probability distribution models this scenario? Is it continuous or discrete?
- (b, 8 pts) Calculate the probability that more than 3 people will suffer using the Poisson distribution as an approximation. (You may leave your answer in terms of exponential terms).

Suppose a rogue boat dealer sells boats which will experience a crack anywhere between 0 and 3 months after purchase with equal probability.

- (c, 2 pts) Which probability distribution models this scenario? Is it continuous or discrete?
- (d, 8 pts) Calculate the probability that the boat will crack in the first 40 days.

Problem II, 20 pts

Find the probability of getting between 3 and 6 heads (inclusive) in 10 tosses of a fair coin, by using the:

- (a, 8 pts) Binomial distribution.
- (b, 12 pts) The normal approximation to the binomial distribution.

Problem III, 20 pts

Upon graduating you get a good job and buy a new Tesla car with a new battery system. Suppose we know that the length of time between full charges of your new Tesla car follows a normal distribution (depending on your driving) with mean 72 hours and standard deviation 24 hours.

- (a, 10 pts) Find the probability that following a full charge, your Tesla will need to be recharged again between 40 and 60 hours of driving.

Now suppose you went to a forum of Tesla owners and asked them how long they drive between recharges. From a sample size of 100 owners, you obtained a mean driving time of $\bar{x} = 65$ hours between charges and a standard deviation of 20 hours. Take the sample standard deviation as an estimate of the population standard deviation σ . (That is, assume as though the population standard deviation is known).

- (b, 10 pts) Find the probability that if you estimate the population mean μ by the sample mean \bar{x} , the absolute estimation error will be less than 7 hours. Please show your work.

Problem IV, 20 pts

Suppose the following probability distribution of scratches on a new teapot is reported by the manufacturer:

$$P(X = 0) = 0.70, \quad P(X = 1) = 0.20, \quad P(X = 2) = 0.07, \quad P(X = 3) = 0.03$$

where X is a random variable counting the number of scratches on the teapot.

- (a, 5 pts) Find the expected value $E[X]$.
- (b, 5 pts) Find the variance $\sigma^2[X]$.

Suppose 50 teapots are bought by a collector.

- (c, 10 pts) Find the probability that these pots have collectively more than 50 scratches. Justify your calculations.

Problem V, 20 pts

Suppose a drug company is testing a new blood pressure drug. Five patients have been sampled and reported the following changes in body temperature (in degrees F) after using the drug:

$$\{1.5, 5.5, 3.6, 3.5, -0.5\}$$

Assume that the population distribution of temperature increases following the use of the drug is roughly bell shaped.

- (a, 5 pts) Find the mean and standard deviation of this sample.
- (b, 15 pts) Using the given information, find the 90 percent confidence interval for the mean change in body temperature associated with the new drug for all patients in the population. Interpret in words the interval that you obtain.