Exam 1 solutions

(1) (a) sample fish via catch and release at random locations in the lake.

(b)

$$\text{sort}(x) = \{\overset{1}{2}, \overset{2}{5}, \overset{3}{6}, \overset{4}{6}, \overset{5}{7}, \overset{6}{7}, \overset{7}{7}, \overset{8}{8}, \overset{9}{8}, \overset{10}{8}, \overset{11}{18}\}$$

$\min = 2$, $\max = 18$

$Q_1 = 25^{th}$ percentile; $L_{25} = \left(\frac{25}{100}\right) 11 = \frac{11}{4} = 2.75$

$\Rightarrow Q_1 = [\text{sort}(x)]_3 = 6$

$Q_2 = 50^{th}$ percentile; $L_{50} = \left(\frac{50}{100}\right) 11 = \frac{11}{2} = 5.5$

$\Rightarrow Q_2 = [\text{sort}(x)]_6 = 7$ = median

$Q_3 = 75^{th}$ percentile; $L_{75} = \left(\frac{75}{100}\right) 11 = 8.25$

$\Rightarrow Q_3 = [\text{sord}(x)]_9 = 8$

$\{\min, Q_1, Q_2, Q_3, \max\} = \{2, 6, 7, 8, 18\}$

$IQR = 8-6=2$

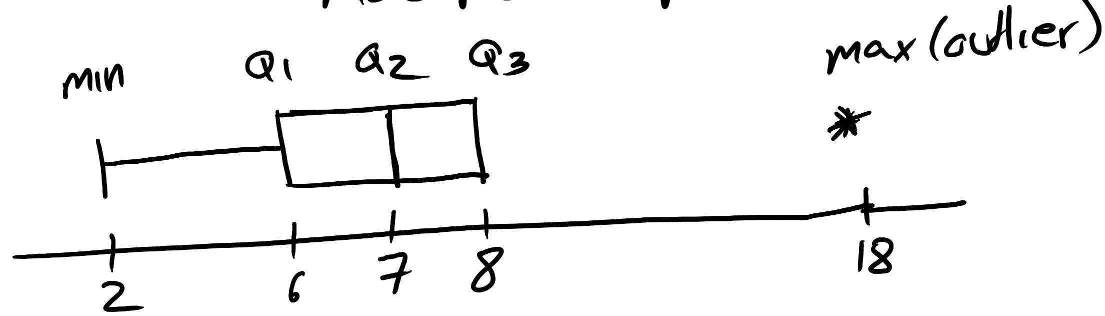
$1.5 \times IQR = 3$

$Q_3 + 1.5 \times IQR = 8+3 = 11$

$Q_1 - 1.5 \times IQR = 6-3=2$

only 18 is outside the interval $[2, 11]$ from the data set so 18 is an <u>outlier</u>.

modified boxplot:



(2)

class	frequency
900-999	1
1000-1099	10
1100-1199	4
1200-1299	3
1300-1399	1
1400-1499	1

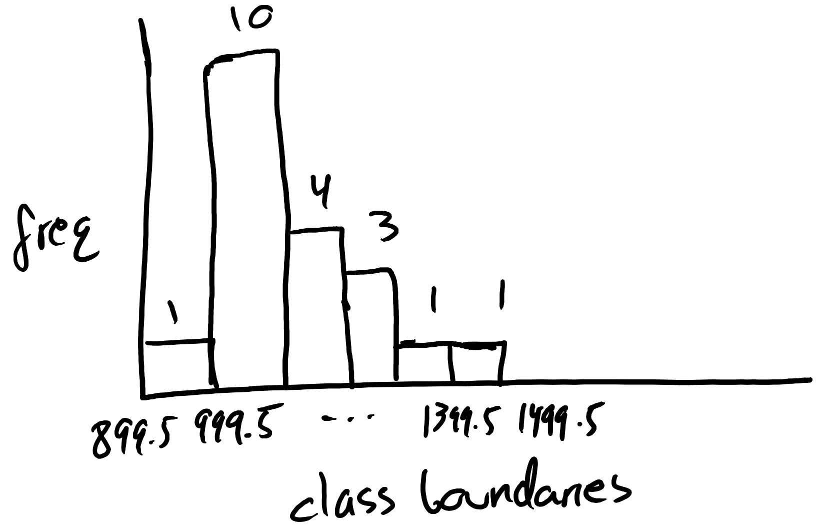
boundries:

899.5-999.5

⋮

1399.5-1499.5

class width $= 100$



Distribution is not approximately bell shaped, it is skewed to the right.

(3) $d = \{3, 2, 4, 2, 3, 2, 4, 4\}$

$\text{sort}(d) = \{2, 2, 2, 3, 3, 4, 4, 4\}$

$\text{mean} = \frac{1}{n} \sum_i d_i = \frac{1}{8}(24) = 3$

$\text{median} = Q_2 = 3 \quad \left(L_{50} = \frac{1}{2} \cdot 8 = 4 \Rightarrow Q_2 = \frac{3+3}{2} = 3 \right)$

mode: 2 and 4 (3 appears only twice)

(b) $s^2 = \dfrac{\sum (x - \bar{x})^2}{n-1} =$

$= \frac{1}{7} \left[(2-3)^2 + (2-3)^2 + (2-3)^2 + (3-3)^2 + (3-3)^2 + (4-3)^2 + (4-3)^2 + (4-3)^2 \right] = \frac{6}{7}$

$$\Rightarrow s = \sqrt{\frac{6}{7}}$$

(c) z score for 10 slices:

$$z = \frac{x - \bar{x}}{\sigma} \quad (\text{take } \sigma = s \text{ as approximation})$$

$$\Rightarrow z = \frac{10-3}{\sqrt{\frac{6}{7}}} = \frac{7}{\sqrt{\frac{6}{7}}} > 2$$

Hence person who ate 10 slices is an outlier.

(d) By the empirical rule, 95% of population values should be in the interval $(\bar{x} - 2\sigma, \bar{x} + 2\sigma)$

$$= \left(3 - 2\frac{7}{\sqrt{\frac{6}{7}}}, 3 + 2\frac{7}{\sqrt{\frac{6}{7}}}\right)$$

(4) $S \cap A$ represents the event of someone being sighted and your friend attacked by the bear.

$A|S$ represents the event of your friend being attacked by bear given someone has been sighted. $P(A|S)$ is the probability of event $A|S$.

$$P(A|S) = \frac{P(A \cap S)}{P(S)} = \frac{0.05}{0.1} = 0.5$$

$P(A|S) \neq P(A)$ so the events are <u>dependent</u>

(5) (a) $P(\text{3 of clubs}) = \frac{1}{52}$ (one card)

(b) $P(\text{not hearts}) = \frac{52-13}{52} = \frac{39}{52}$

(c) $P(\text{ten or a spade})$

notice 10 of spades is included here

use $P(A \cup B) = \underbrace{P(A)}_{10} + \underbrace{P(B)}_{\text{spade}} - \underbrace{P(A \cap B)}_{\text{10 of spades}}$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

(d) $P(\text{heads on last two flips}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(independent events)

(e) $P(A \cap B') = ?$

use that:

$$(A \cap B') \cup (A \cap B) = A \quad \text{and} \quad (A \cap B') \cap (A \cap B) = \phi$$

empty set

$$P[(A \cap B') \cup (A \cap B)] = P[A \cap B']$$

$$+ P[A \cap B] - P[\phi] = P[A]$$

$$\Rightarrow P[A \cap B'] = P[A] - P[A \cap B]$$

$$= \frac{1}{2} - \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

use De' Morgan's rule:

(f) $$P[A' \cap B'] = P[(A \cup B)'] =$$

$$= 1 - P[A \cup B]$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{6}\right] = 1 - \left[\frac{3}{6} + \frac{2}{6} - \frac{1}{6}\right]$$

$$= \frac{6}{6} - \frac{4}{6} = \frac{2}{6} = \frac{1}{3}$$

Above calculations can be simplified if you argue that A and B are independent.