MATH 21-01 (Introductory Statistics, Voronin, S.), Exam #1 (100 points).

No electronic devices or books/notes. Please show your work for all problems. Clearly state your answer to each question. Sufficient written reasoning and calculation work is required to obtain full credit. Partial credit will be awarded where possible. Start: 9:30 AM. End: 10:20 AM. October 11th, 2016.

Problem I, 20 pts

During a senior year Spring hike you discover a large lake on the outskirts of Tufts. You observe many fish swimming in the lake and attempt to statistically analyze their weights by obtaining a small sample but representative sample.

- (a, 2 pts) Describe a way of obtaining a good sample of the fish in order to draw conclusions about the population.

Suppose that you have caught and measured 11 fish. We call this sample X. The weights in pounds are as follows: $X = [7, 7, 6, 5, 8, 8, 2, 18, 6, 8, 7]$.

- (b, 7 pts) Find all the components of the five number summary of X above.
- (c, 4 pts) Find the IQR. State the IQR characterization for outliers. Are there any outliers in the weights of the fish above? Justify your answer.
- (d, 7 pts) Draw a modified boxplot of the data in X. Clearly label the different parts of your plot.

Problem II, 15 pts

Given the following list of brain volumes (in cm^3) given below:

$$1005, 963, 1035, 1027, 1281, 1272, 1051, 1079, 1034, 1070,$$
$$1173, 1079, 1067, 1104, 1347, 1439, 1029, 1100, 1204, 1160$$

- (a, 7 pts) Construct and label a frequency distribution for the above data, given that the first class has a lower class limit of 900 and a class width of 100.
- (b, 8 pts) From your frequency distribution, construct and label a histogram for the data. The x-axis of the histogram should be marked with the class boundaries. Is the distribution approximately bell shaped? Is it skewed towards a particular direction?

Problem III, 25 pts

Suppose, a selection of Tufts students is sampled and asked about how many pizza slices they have eaten in the past week. The results are:

$$3, 2, 4, 2, 3, 2, 4, 4$$

In the following questions, you can leave your results in terms of fractions and square roots, as necessary.

- (a, 8 pts) What are the mean, mode, and median of pizza slices eaten?
- (b, 8 pts) What are the standard deviation and variance of this sample?
- (c, 4 pts) Suppose another Tufts student is sampled who has eaten 10 slices. What's his approximate z-score? Is he an outlier according to the z-score outlier characterization?
- (d, 5 pts) If we use the mean and std deviation of the sample as estimators for the population and the population is normally distributed, write down the interval of pizza slices eaten in which approximately 95% of the population measurements are expected to lie.

Problem IV, 10 pts

A few black bears have made their way from Boston Common to Tufts campus, prior to hibernation. Suppose S is the event of your friend being sighted by a bear on Tufts campus. Suppose A is the event of a bear attacking someone it sights. Let $P(S) = 0.1$, $P(A) = 0.2$, and $P(S \cap A) = 0.05$.

- (a, 3 pts) What does $P(S \cap A)$ represent? What does $P(A|S)$ represent?
- (b, 7 pts) Calculate $P(A|S)$. Are A and S independent?

Problem V, 30 pts

A card is drawn at random from an ordinary deck of 52 cards. Find the probability that the card is:

- (a, 5 pts) A three of clubs or a six of diamonds.
- (b, 5 pts) Of any suit except for hearts.
- (c, 5 pts) A ten or a spade.

After the card is drawn, a fair coin is flipped ten times.

- (d, 5 pts) If the first six flips of the coin were tails and the next two heads, what is the probability of getting heads on the last two flips?

Suppose for some events A and B, different from those above, the following probabilities hold: $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{6}$.

- (e, 5 pts) Find $P(A \cap B')$.
- (f, 5 pts) Find $P(A' \cap B')$.

Some formulas

sample std deviation $\quad s = \sqrt{\frac{\sum_{k=1}^{n}(x_k - \bar{x})^2}{n-1}}$

set relation I : $(A \cap B') \cup (A \cap B) = A$

set relation II : $(A \cap B') \cap (A \cap B) = \emptyset$

set relation III : $(A' \cap B') = (A \cup B)'$

Card deck: 52 card in four suits (clubs, diamonds, hearts, spades). 13 cards of each suit.